A Rotational Stiffness Model for Magnetic Microrobots in Soft Tissue

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INTRODUCTION

Magnetic-screw microrobots capable of traversing soft tissue are one of the longest-studied microrobots [1]. They comprise a screw and an attached/embedded permanent magnet with magnetic dipole $\boldsymbol{m}$ (units A·m^2) with its magnetization orthogonal to the screw axis [Fig. 1(a)]. They are simultaneously propelled and steered by a magnetic field $\boldsymbol{b}$ (units T) that is rotated about, and orthogonal to, some axis $\boldsymbol{\omega}$, resulting in an instantaneous applied torque $\boldsymbol{\tau} = \boldsymbol{m} \times \boldsymbol{b}$ (units N·m). They are similar to magnetic helical microrobots that swim through fluid, which have received more attention. However, the differences between soft tissue and fluid makes the dynamics of these microrobots distinct. It has traditionally been assumed that $\boldsymbol{\omega}$ represents the desired steering direction, with the goal of causing the screw's axis $\boldsymbol{a}$ to align with $\boldsymbol{\omega}$ over time. This alignment often happens, but not always, and even when it does happen the screw often does not take the shortest path (i.e., by $\boldsymbol{a}$ moving through the plane spanned by $\boldsymbol{a}$ and $\boldsymbol{\omega}$). Prior modeling of these microrobots in soft tissue has assumed this steering model, and the resulting models have had limited predictive power [2], requiring further characterization of the deviations from what would be expected [3].

Our group is moving away from the notion that the goal in steering is to align $\boldsymbol{a}$ with $\boldsymbol{\omega}$, and instead simply modeling the microrobot motion that results due to a given rotating magnetic field; this model can then be used subsequently by a controller or motion planner. Steering a magnetic screw in soft tissue is accomplished by the screw pitching/yawing with respect to the world frame, stressing the soft tissue in the process, and then advancing within the stressed tissue. We are interested in the effective rotational stiffness of the soft tissue for a given microrobot. That is, for a given applied torque $\boldsymbol{\tau}$ orthogonal to $\boldsymbol{a}$, what is the angular deflection of $\boldsymbol{a}$ with respect to the world frame? In general, we expect the angular deflection θ (units rad, effectively dimensionless) to be a function of five independent variables: the length L (units m) and diameter D (units m) of the magnetic screw, the modulus of elasticity E (units Pa = N·m^{-2}) and Poisson's ratio ν (dimensionless) of the soft tissue, and the applied torque $\tau = \|\boldsymbol{\tau}\|$ (units N·m) orthogonal to $\boldsymbol{a}$. Given these six variables and two dimensions {m, N}, the Buckingham Π theorem [4] tells us that the physics can be described by just $6 - 2 = 4$ dimensionless variables, leading to a function of the form:

$$\theta = f\left(\frac{L}{D}, \frac{\tau}{ED^3}, \nu\right). \tag{1}$$

In addition, because all soft tissues have a Poisson's ratio of $\nu \approx 0.5$ [5], the number of independent inputs is effectively reduced by one. In this paper, we describe an empirical study in which we characterize this function, which generalizes across scale and tissue type.

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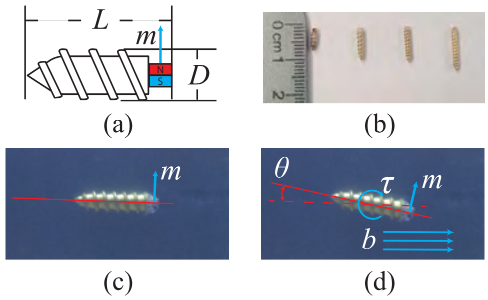

Fig. 1 (a) Schematic of a magnetic-screw microrobot with definition of dimensions, (b) fabricated magnetic-screws microrobots in this study, and side-view images of a magnetic-screw microrobot embedded in a soft-tissue phantom (c) before and (d) after applied magnetic field $\boldsymbol{b}$ causes rotation θ due to torque $\boldsymbol{\tau} = \boldsymbol{m} \times \boldsymbol{b}$.

MATERIALS AND METHODS

Microrobots were fabricated from #2 (for $L/D \approx \{3, 4\}$) and #3 (for $L/D \approx \{5, 6\}$) brass wood screws. The back of each screw was cut, ground, and filed down, and a 1 mm^3 N50-grade NdFeB permanent magnet was affixed to it, using cyanoacrylate, oriented such that its dipole was diametrically aligned. The fabricated microrobots are shown in Fig. 1(b). We measured the final dimensions [see Fig. 1(a)] as $L = \{6.6, 9.1, 10.2, 12.5\}$ mm and $D = \{2.2, 2.3, 2.0, 2.0\}$ mm, respectively, resulting in $L/D = \{3.0, 4.0, 5.1, 6.3\}$. The magnets were measured to have side lengths of $\{1.02, 1.03, 0.95, 0.98\}$ mm, respectively, which were used to approximate each magnet's volume. The volumes were then multiplied by the N-50-grade magnetization of 1.14×10^6 A/m to calculate $\|\boldsymbol{m}\|$.

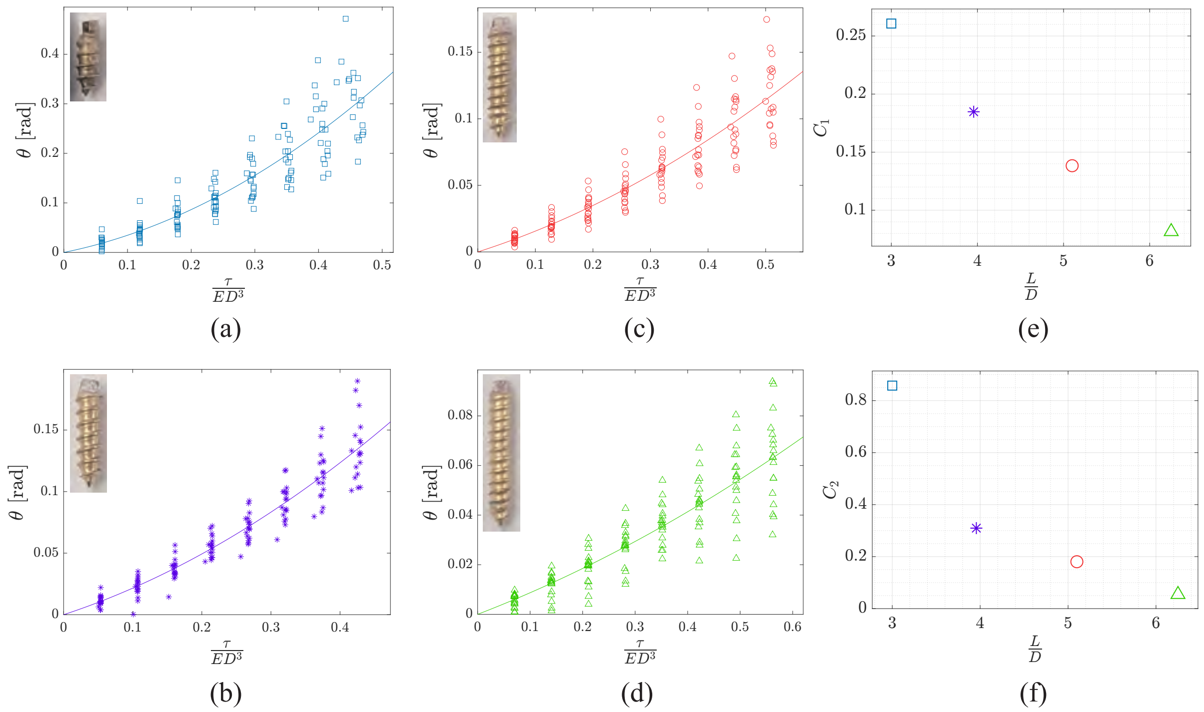

Fig. 2 Experimental results of θ vs. $\tau/(ED^3)$ for (a) $L/D = 3.0$, (b) $L/D = 4.0$, (c) $L/D = 5.1$, and (d) $L/D = 6.3$; insets show the microrobots used. The coefficients C_1 and C_2 of the least-squares model are shown in (e) and (f), respectively.

For a soft-tissue phantom, we made blocks of agar gel. We used an agar concentration of 0.325 wt%, for which $E = 1.9 \times 10^3$ Pa [6]. Agar blocks were cut and inserted into 15.6 mm × 44.7 mm × 38.4 mm ($L \times W \times H$, internal dimensions) acrylic test chambers. A microrobot was then inserted into the test chamber via an access port on the side of the chamber. The test chambers were then inserted into the central workspace of tri-axial Helmholtz coils (see [2], [3] for details of the coils used).

For each trial, the screw was driven into a block of gel such that the screw axis was approximately horizontal and parallel with the axis of the inner Helmholtz coil, and such that the magnetic dipole was approximately vertical [Fig. 1(c)]. Then the field of the inner coil was commanded to values of $\|\boldsymbol{b}\| \in [1, 8]$ mT in increments of 1 mT, causing a rotation of the microrobot axis $\boldsymbol{a}$ by an angle θ [Fig. 1(d)]. This process was repeated in four blocks of agar, at three locations per block, for each microrobot. In postprocessing, the applied torque was calculated as $\tau = \|\boldsymbol{m}\|\|\boldsymbol{b}\| \cos\phi$, where ϕ was the observed angle of $\boldsymbol{a}$ with respect to horizontal.

RESULTS

In Figs. 2(a)–2(d), we show the results for angular rotation θ as a function of $\tau/(ED^3)$ for each of our L/D ratios. We observed that empirical functions of the form

$$\theta = C_1 \left(\frac{\tau}{ED^3}\right) + C_2 \left(\frac{\tau}{ED^3}\right)^2 \tag{2}$$

fit using least-squares regression capture the mean trend of the data, with no clear trend remaining in the residuals. We provide the values for C_1 and C_2 in Figs. 2(e)–2(f), respectively; these can be interpolated to approximate the coefficients for any L/D value in the range considered.

The model naturally leads to a Hooke's law rotational stiffness K (units N·m/rad), valid for small deflections:

$$\frac{1}{K} = \left.\frac{\partial\theta}{\partial\tau}\right|_{\tau\to 0} \quad\Rightarrow\quad K = \frac{ED^3}{C_1} \tag{3}$$

DISCUSSION

The results of this study are generally applicable to rod-shaped microrobots. They are also somewhat applicable to a new class of screw-tip soft magnetically steerable needles [6]; however, because of the continuum structure of these devices, we would expect the angular deflection to be less than predicted by the model herein.

REFERENCES

[1] B. J. Nelson, I. K. Kaliakatsos, and J. J. Abbott, "Microrobots for minimally invasive medicine," *Annu. Rev. Biomed. Eng.*, vol. 12, pp. 55–85, 2010.

[2] N. D. Nelson, J. Delacenserie, and J. J. Abbott, "An empirical study of the role of magnetic, geometric, and tissue properties on the turning radius of magnetically driven screws," in *Proc. IEEE Int. Conf. Robot. Autom.*, 2013, pp. 5352–5357.

[3] A. W. Mahoney, N. D. Nelson, E. M. Parsons, and J. J. Abbott, "Non-ideal behaviors of magnetically driven screws in soft tissue," in *Proc. IEEE/RSJ Int. Conf. Intell. Robot. Syst.*, 2012, pp. 3559–3564.

[4] E. Buckingham, "On physically similar systems; illustrations of the use of dimensional equations," *Phys. Rev.*, vol. 4, no. 4, p. 345, 1914.

[5] D. Jastrzebski, *Nature and Properties of Engineering Materials*. John Wiley & Sons, 1987.

[6] A. J. Sperry, T. J. Schwehr, E. K. Pinegar, O. B. Richards, J. D. Rolston, M. D. Alexander, B. Coats, J. J. Abbott, and A. Kuntz, "Screw-tip soft magnetically steerable needles," *IEEE Trans. Med. Robot. Bionics*, vol. 6, no. 1, pp. 4–17, 2024.