Managing magnetic force applied to a magnetic device by a rotating dipole field

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We demonstrate that the attractive magnetic force acting on a rotating magnetic device (e.g., a magnetic microrobot), actuated using a rotating magnet dipole, can be converted into a lateral force by rotating the actuator dipole according to a specific open-loop trajectory. Results show rotating magnetic devices can be rolled and simultaneously pushed along a surface by the lateral force, resulting in significant increase in velocity. We also demonstrate that the lateral force magnitude can be sufficient to levitate the magnetic device. The results apply to rotating magnetic devices of any size provided inertia has a negligible contribution to its dynamics. © *2011 American Institute of Physics*. [doi:10.1063/1.3644021]

Untethered robots at the micro and mesoscale have become an active area of research because of their potential impact to minimally invasive medicine.[1] Devices fabricated with a magnetic component, on which forces and torques are applied by an external magnetic field, are demonstrating particular promise. Approaches to magnetic locomotion include pulling using magnetic forces and those, where the primary mode of operation is rotation, such as helical propulsion and rolling. In the case of rotating devices using a single rotating permanent magnet as the field source,[2–4] the magnetic dipole field generates torque, which causes the device to rotate while simultaneously generating a magnetic force that typically tends to attract the device toward the permanent magnet (Figs. 1(a) and 1(c)).

For *in vivo* medical applications, an attractive force too large in magnitude may cause the magnetic device to pull toward the actuator, resulting in tissue deformation and potentially trauma. The problem may become self-compounding as the magnitude of the attractive force increases dramatically with decreasing distance from the magnetic actuator. For practical use, the attractive force must be manageable, and in some cases it may need to be substantially eliminated. In this study, we demonstrate that if the actuating permanent magnet is driven according to a specific open-loop rotation trajectory of nonconstant speed, the attractive magnetic force acting on a magnetic device vanishes and is converted into a lateral force nearly constant in magnitude. This lateral force may oppose or contribute to the rolling of the device on a surface (Figs. 1(b) and 1(d), respectively), and it can be large enough in magnitude to overcome the device's weight when oriented against the gravity. This study presents theoretical and experimental analysis of this phenomenon. The results apply to any rotating magnetic device ranging in size from the microscale (e.g., a magnetic microrobot) to the mesoscale (e.g., a magnetic capsule endoscope), actuated by a single rotating permanent magnet, provided that inertial effects are negligible. Such devices may roll on a surface[2,3] or employ a helix to generate forward motion out of rotation.[4,5]

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Define a stationary world frame with axes $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$. Assuming that the applied magnetic field $\mathbf{B}$ is generated using a single magnetic dipole (referred to as the actuator magnet) with dipole moment $\mathbf{M}$ that can be accurately approximated using the point-dipole model, then $\mathbf{B}$ at the position of the magnetic device, described relative to the center of the actuator magnet with vector $\mathbf{p}$, is

$$\mathbf{B} = \frac{\mu_0}{4\pi|\mathbf{p}|^3}\left[3\frac{\mathbf{p}\mathbf{p}^T}{|\mathbf{p}|^2} - \mathbb{I}\right]\mathbf{M} = \frac{\mu_0|\mathbf{M}|}{4\pi|\mathbf{p}|^3}\begin{bmatrix} 0 \\ \sin(\theta) \\ 2\cos(\theta)\end{bmatrix}, \quad (1)$$

where μ_0 is the permeability of free space constant, $\mathbb{I}$ is the identity matrix, and θ describes a specific parameterization shown in Fig. 2. Without loss of generality, the rightmost equality in Eq. (1) constrains $\mathbf{p}$ to lie on the $\mathbf{z}$ world axis with the actuator rotating around the $\mathbf{x}$ world axis, constraining $\mathbf{M}$ to the $\mathbf{y}$-$\mathbf{z}$ plane.

The magnetic torque $\boldsymbol{\tau}$ produced on a dipole moment $\mathbf{m}$ (i.e., the moment of the magnetic body attached to the rotating device) by the applied field $\mathbf{B}$ causes $\mathbf{m}$ to align with $\mathbf{B}$ and is expressed by $\boldsymbol{\tau} = \mathbf{m} \times \mathbf{B}$. The magnetic torque causes the device to rotate in the opposite rotation direction of the actuator $\mathbf{M}$. We model the magnetic device's rotational dynamics rolling on a surface or rotating in a fluid or lumen using the applied magnetic torque and a linear drag torque with coefficient c, while assuming that the torque due to the device's inertia is negligible. If ϕ measures the angle of $\mathbf{m}$ from the $\mathbf{z}$ axis, then the device's rotational dynamics are

$$-c\dot{\phi} + |\mathbf{m}||\mathbf{B}|\sin(\alpha) = 0, \quad (2)$$

implying that $\dot{\phi}$ is linearly coupled to the rotating actuator by the magnetic torque, which is maximized when $\alpha = 90°$. By examining the conditions when $\alpha = 90°$ at steady-state, according to Eq. (2), we find that the actuator must be driven so that its rotation velocity $\dot{\theta}$ follows

$$\dot{\theta} = \frac{\mu_0|\mathbf{m}||\mathbf{M}|}{8\pi|\mathbf{p}|^3 c}\left(1 + 3\cos^2(\theta)\right)^{\frac{3}{2}} = K\left(1 + 3\cos^2(\theta)\right)^{\frac{3}{2}}. \quad (3)$$

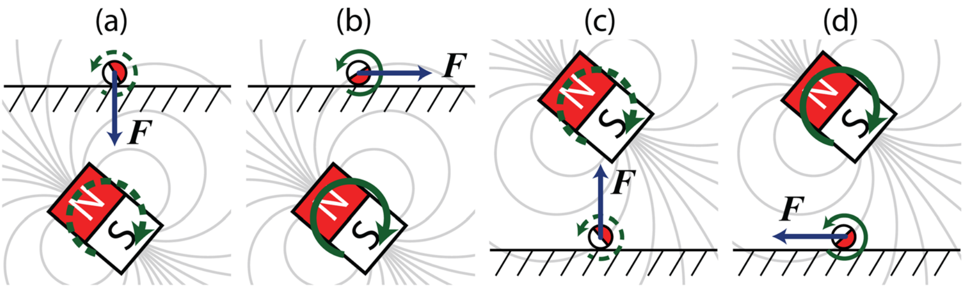

FIG. 1. (Color online) When rotating the actuator quasistatically, the magnetic force tends to attract the magnetic device (e.g., a microrobot) toward the actuator while causing it to roll (a), (c). Operating the actuator dynamically as described herein causes the magnetic force to oppose the magnetic device's rolling motion in the case where the rolling surface lies between the rotating actuator and the device (b) and to contribute to rolling otherwise (d).

Driving the actuator according to Eq. (3) stably maintains $\alpha = 90°$; it can be shown that $\alpha \to 90°$ as $t \to \infty$ for any initial conditions of α and θ. Although the magnetic force acting on the device influences the coefficient c, significant effects to the rotational behavior of the device actuated in this paper are not observed.

Eq. (3) requires the actuator's orientation θ, the device's position $\mathbf{p}$, and the speed coefficient K to be known but does not require measurement of the device's magnetized orientation, which can be difficult for some devices such as microrobots of spherical or cylindrical (polarized diametrically) geometry. In practice, θ is known, the device's position $\mathbf{p}$ can be measured using a variety of methods such as computer vision or medical imaging, and an estimate $\hat{K}$ is used in place of K. If $\hat{K} \leq K$, then it can be shown that α converges to $\sin^{-1}(\hat{K}/K)$. If $\hat{K} > K$, no steady-state α exists and the device will step out of synchronization with the actuator. $\hat{K}$ can be measured by incrementally increasing $\hat{K}$ until the device is observed to step out of synchronization with the actuator, at which time $\hat{K} \approx K$.

When the magnetic device is positioned as in Fig. 2 and the actuator is rotating according to Eq. (3), making the device's dipole moment $\mathbf{m}$ lag the applied field $\mathbf{B}$ by 90°, then we find that the magnetic force $\mathbf{F} = (\mathbf{m} \cdot \nabla)\mathbf{B}$ acting on the device, using Eq. (1), is

$$\mathbf{F} = \frac{3\mu_0|\mathbf{m}||\mathbf{M}|}{4\pi|\mathbf{p}|^4}\left(\frac{1+\cos^2(\theta)}{\sqrt{1+3\cos^2(\theta)}}\right)\begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}. \qquad (4)$$

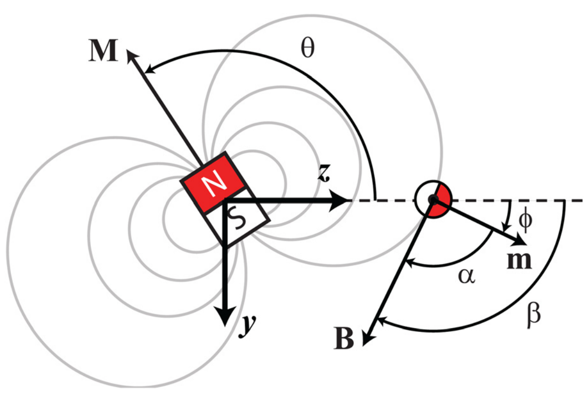

FIG. 2. (Color online) The magnetic device is positioned on the **z** axis and the actuator magnet rotates around the **x** axis (out of the image), constraining the actuator and device dipole moments, **M** and **m**, respectively, to the **y**-**z** plane.

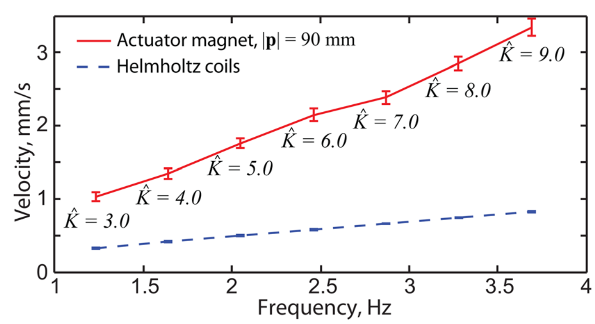

FIG. 3. (Color online) Rolling velocity of the magnetic device as a function of rotation frequency obtained with the actuator positioned 90 mm above the device (Fig. 1(d)) and using a triaxial Helmholtz coil system.[5] Each data point is the average of four trials, and the error bars denote one standard deviation.

When the actuator rotates in the positive direction while satisfying Eq. (3), the applied magnetic force lies in the negative $\mathbf{y}$ direction. In general, if $\boldsymbol{\Omega}$ represents the actuator's angular velocity vector, then the magnetic force points in the direction of $\boldsymbol{\Omega} \times \mathbf{p}$. No component of the magnetic force attracts the device to the actuator. The magnitude $|\mathbf{F}|$ in Eq. (4) varies from 94.3% to 100% of $\frac{3\mu_0|\mathbf{m}||\mathbf{M}|}{4\pi|\mathbf{p}|^4}$ as θ changes, making the magnetic force nearly constant in magnitude.

The method presented herein was verified experimentally using a magnetic device consisting of a diametrically polarized NdFeB (Grade N42) cylindrical magnet 6.35 mm in length and 3.17 mm in diameter, weighing 1.2 g. The

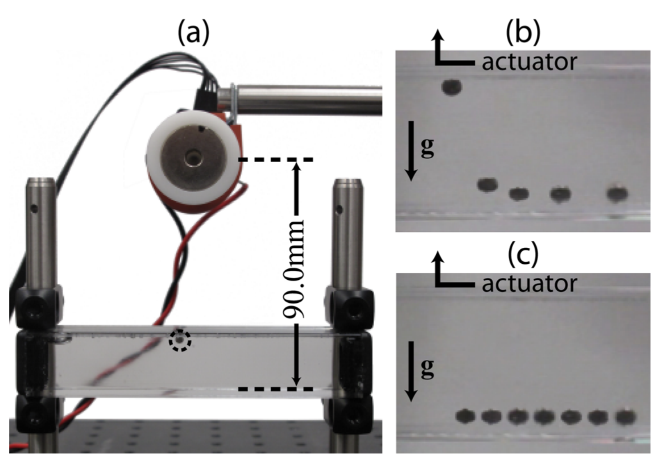

FIG. 4. (Color online) An experimental setup (a) with the magnetic device circled. Image sequences show the device driven right to left using 1.23 Hz actuation with (b) a constant angular velocity and (c) according to Eq. (3) with $\hat{K} = 3.0$. Use of Eq. (3) significantly reduces the attractive magnetic force (enhanced online). [URL: http://dx.doi.org/10.1063/1.3644021.1]

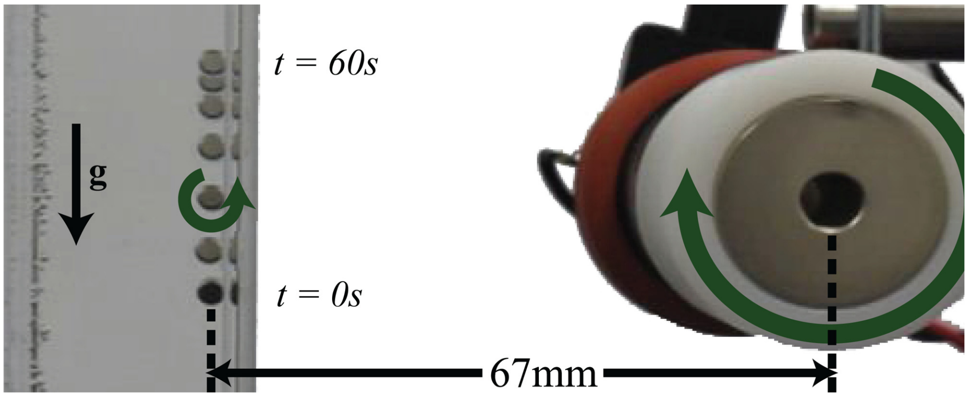

FIG. 5. (Color online) Image sequence shows the magnetic force, the subject of this work, levitating the magnetic device against both its weight and the rolling force. The image sequence begins at $t = 0$ s where the device is at static equilibrium and rises 24 mm to a dynamic equilibrium at $t = 60$ s with the actuator rotation satisfying Eq. (3) for $\hat{K} = 20$. Images are shown in 10 s increments (enhanced online). [URL: http://dx.doi.org/10.1063/1.3644021.2]

device was actuated using magnetic fields produced by a diametrically polarized NdFeB (Grade N42) cylindrical magnet 25.4 mm in length and diameter with magnetic moment $|\mathbf{M}| = 12.6\ \mathrm{A \cdot m^2}$, driven by a Maxon 24 V A-Max DC motor with an Advanced Motion Controls servo control drive and amplifier. For any $\hat{K}$, the actuator's instantaneous rotation speed (given by Eq. (3)) varies with the actuator's orientation from $\hat{K}$ to $8\hat{K}$ throughout each cycle with an effective peak-to-peak frequency $f \approx 0.41\hat{K}$ Hz. To remain within the torque constraints of the motor while demonstrating the effects of applied force, the drag coefficient c was increased by immersing the magnetic device in a rectangular acrylic tank of corn syrup. A triaxial Helmholtz coil system,[5] which generates uniform magnetic fields and force-free magnetic torque, is used for experimental comparison.

When $\hat{K} < K$ and $\alpha < 90°$ at steady-state, we find numerically that the horizontal and attractive components of the magnetic force are nonzero and fluctuate with θ. As $\hat{K} \to K$, the average lateral and attractive components over one actuator cycle increase and decrease, respectively, until the magnetic force converges to Eq. (4) when $\hat{K} = K$. Fig. 3 shows the horizontal device rolling velocity when positioned as in Fig. 1(d) and actuated according to Eq. (3) with increasing $\hat{K}$. Fig. 3 also shows the horizontal rolling velocity when actuated within the Helmholtz system at equivalent frequencies for comparison. The Helmholtz system applies negligible force; therefore, the significant increase in velocity when driven with the actuator magnet is attributable to the horizontal magnetic force that is the subject of this work. As $\hat{K}$ increases, the horizontal magnetic force and its contribution to the horizontal velocity increase as shown. As $|\mathbf{p}|$ increases, we expect the contribution of the applied force will diminish as $|\mathbf{p}|^{-4}$, making the velocity approach that in the Helmholtz system.

Previous approaches consider quasistatic actuation using a single rotating permanent magnet at constant frequencies where the lead angle α is small.[2–4] When operated in this manner, the magnetic force always attracts the magnetic device toward the actuator magnet. Operating the actuator according to Eq. (3) reduces the attractive force even at equivalent frequencies. Figs. 4(b) and 4(c) show an image sequence of the device operated by rotating the actuator at a constant angular velocity of 1.23 Hz compared to rotating the actuator according to Eq. (3) with the same frequency. With a constant angular velocity, although the magnetic force initially pulls the device in the rolling direction at 1.67 mm/s, it decelerates as the magnetic force transitions upward and overcomes the device's weight, attracting the device toward the actuator. Driving the actuator according to Eq. (3) reduces the attractive magnetic force, and the device remains on the lower surface while traveling at 0.78 mm/s.

Aside from managing the magnetic force in a manner that simultaneously reduces the attractive component and contributes to the device's rolling velocity, the magnetic force can also be used for levitation. Fig. 5 shows an image sequence with the device configured according to Fig. 1(b) (rotated 90°). The device begins at $t = 0$ s in its static equilibrium position with no actuator rotation and rises 24 mm to its dynamic equilibrium position at $t = 60$ s when actuated according to Eq. (3) with $\hat{K} = 20$. When the actuator rotates clockwise, the device rotates counterclockwise producing a rolling force in the same direction as gravity. The component of the applied magnetic force causing the device to levitate, therefore, opposes both the rolling force and the device's weight, resulting in a net vertical motion, while the remaining attractive component pins the device to the wall.

We have demonstrated theoretically and experimentally that the attractive magnetic force acting on a rotating magnetic device can be diminished and converted into a lateral force when positioned as in Fig. 2 and actuated according to Eq. (3) in an open-loop fashion. Such operation can be used to simultaneously roll and push a device on a surface, resulting in potentially significant increases in rolling velocity, or it may be used for device levitation. Although the experiments were performed on a millimeter-scale device, the results of this work apply to rotating magnetic devices of any size with negligible inertia actuated with a single rotating magnetic dipole.

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