

Electromagnetics

Optimization of Coreless Electromagnets to Maximize Field Generation for Magnetic Manipulation Systems

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Abstract—Magnetic manipulation systems typically take the form of a set of stationary electromagnets surrounding, and projecting their fields into, a common workspace. The electromagnets are typically wound with cylindrical geometries, sometimes with a tapered end for tighter packing. Motivating this study was the conjecture that an optimal general electromagnet designed with minimal constraints on the allowable geometry would outperform an optimal (tapered) cylindrical electromagnet in terms of maximizing field generation at a given point on the axis of the electromagnet for a given amount of power. In this letter, we provide a method to calculate the optimal general geometry in the case of coreless electromagnets, and verify that it is not cylindrical, as expected. However, we also find that the optimal general electromagnet negligibly outperforms the optimal (tapered) cylindrical electromagnet for any parameter set. Because of the added complexity in fabricating an optimal general electromagnet, designers of magnetic manipulation systems using coreless electromagnets can limit their design space by considering only simple (tapered) cylindrical electromagnets, with confidence that their design will be very close to the optimal general design.

Index Terms—Electromagnetics, electromagnetic devices.

I. INTRODUCTION

There has been substantial interest in recent years in the development of magnetic manipulation systems to manipulate objects under the guidance of an optical microscope [Diller 2013, Kratochvil 2014], to manipulate objects *in vivo* for minimally invasive medical procedures [Kummer 2010], and to create new types of haptic interfaces that do not require a mechanical linkage [de Jong 2010, Berkelman 2013, Brink 2014]. These systems typically take the form of a set of stationary electromagnets surrounding their manipulation workspace with the axes of the electromagnets (approximately) pointing toward a common central point [Kummer 2010, Diller 2013, Kratochvil 2014], but other systems have comprised only a single electromagnet [de Jong 2010, Brink 2014] or a planar array of electromagnets with their axes parallel [Berkelman 2013]. The electromagnets may comprise ferromagnetic cores or be coreless (i.e., comprise air cores).

The electromagnets in the previously developed systems are typically wound to have a cylindrical outer geometry, sometimes with a tapered end near the workspace to enable the electromagnets to be placed closer together than would be possible with a simple cylinder. Throughout this letter, we will refer to cylindrical geometries both with and without a tapered end simply as “cylindrical” (see Fig. 1). In the prior works that have aimed to critically compare various magnetic manipulation systems, the use of cylindrical electromagnets has never been questioned [Fisher 2006, Erni 2013]. However, it is easy to show, at least in a coreless electromagnet, that a coil wrap far from the center of the workspace contributes less to the total magnetic field there than does a coil wrap close to the center of the workspace, even though

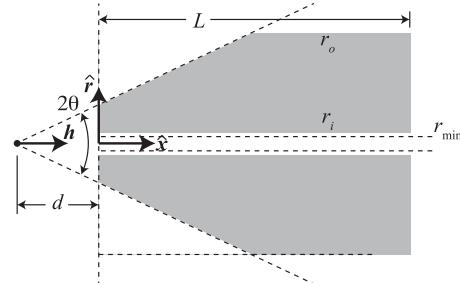


Fig. 1. Coordinate system and design constraints used throughout this letter, as well as definitions of parameters defining a cylindrical electromagnet. We are interested in generating a magnetic field \mathbf{h} at a distance d from one end of the electromagnet. There may be some minimum radius r_{\min} as a design constraint, due to the minimum bending radius of the wire chosen. A cylindrical electromagnet will be defined by an inner radius $r_i \geq r_{\min}$, an outer radius $r_o > r_i$, a length L , and (optionally) a taper defined by the angle θ measured from the \hat{x} -axis.

these two hypothetical wraps may each consume the same amount of electrical power. As such, it was our motivating conjecture that electromagnets with geometries restricted to cylinders are not optimal, in the sense that they do not generate the largest possible field in the central workspace for the electrical power being consumed and the associated heat being generated. Because the field generated by a given electromagnet is linear with respect to current, whereas the heat generated is quadratic with respect to current, electromagnet optimization can have a significant impact in practical system implementations.

In this letter, we provide a method to calculate the optimal general geometry, in the case of coreless electromagnets, which maximizes the magnetic field at a specific remote point on the axis of the electromagnet (e.g., corresponding to the center of the workspace in a magnetic

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manipulation system) for a given amount of conductor (i.e., wire). We employ the same basic methodology employed in Miklavc [1974] and Haignere [1976]. We confirm our hypothesis that optimal cylindrical electromagnets are suboptimal more generally. However, we also find that the optimal general electromagnet negligibly outperforms the optimal cylindrical electromagnet for any parameter set, and we conclude that the benefits of the optimal general electromagnet do not outweigh the increased complexity in manufacturing, relative to a cylindrical electromagnet; this is the principal contribution of our study.

In some prior works, researchers have optimized to balance field generation with power consumption [Gosselin 2004, Yang 2004, Yuan 2015], but in this study we simply assume the volume of conductor is given, as we optimize the shape of the electromagnet. We also choose to optimize based on maximizing field generation at a given point, although one could choose to define “optimal” differently [Morgan 2001, Robertson 2012]. Finally, we do not consider the case of electromagnets with ferromagnetic cores.

II. PROBLEM STATEMENT

We consider an electromagnetic coil in a cylindrically symmetric region, V , obtained by rotating the two-dimensional region $U \subset \mathbb{R}_+^2$ around the \hat{x} -axis. Here, \mathbb{R}_+^2 denotes the first quadrant. The volume of V can be expressed in terms of the two-dimensional set U as

$$v(U) = \iiint_V dV = \iint_U 2\pi r dr dx. \quad (1)$$

This volume can be viewed as the integration of infinitesimal coils of cross-sectional area $dr dx$ and circumference $2\pi r$. We assume a constant current density j circulating around the \hat{x} -axis in the region V . As a consequence of the cylindrical symmetry, we obtain that the magnetic field at the point $\mathbf{p} = (-d, 0, 0)$ is $\mathbf{h}(\mathbf{p}) = h\hat{\mathbf{x}}$, where

$$h(U) = \frac{j}{2} \iint_U f(r, x) r dr dx \quad (2a)$$

$$f(x, r) = \frac{r}{((x + d)^2 + r^2)^{\frac{3}{2}}}. \quad (2b)$$

Here, we think of h as a function of the set U . This is a generalization of the well-known formula for the axial field of a single current loop.

We are interested in finding the coil configuration with a desired volume, v_{des} , that maximizes h (i.e., the magnitude of the magnetic field, \mathbf{h}), at the point \mathbf{p} . This can be formulated as finding the set $U \subset \mathbb{R}_+^2$ that solves the shape optimization problem

$$\max h(U) \quad (3a)$$

$$\text{s.t. } U \subset A \quad (3b)$$

$$v(U) = v_{\text{des}}. \quad (3c)$$

We use the constraint in (3b) to specify an admissible region, $A \subset \mathbb{R}_+^2$, that can be used in the design. This could be a general set, but we will focus on the admissible region illustrated in Fig. 1. We assume that the coils cannot be wrapped at a radius smaller than $r_{\min} \geq 0$, and (in general) they should be contained in a cone region, in which case, we have that $A = A_1$, where

$$A_1 = \{(x, r) : x \geq 0, r_{\min} \leq r \leq (x + d) \tan(\theta)\}. \quad (4)$$

III. GENERAL OPTIMAL SOLUTION

The following theorem gives the solution to the shape-optimization problem (3).

Theorem 1: Provided $v(A) \geq v_{\text{des}}$, there is a unique solution, U^* , to (3) given by

$$U_c = \{(x, r) \in A : f(x, r) \geq c\} \quad (5)$$

for some constant $c \in [0, 2/(3\sqrt{3}d^2)]$ chosen so that $v(U_c) = v_{\text{des}}$. If every horizontal slice (constant r) of A is an interval, then U^* has the same property, i.e., there exist functions $\eta_2(r) \geq \eta_1(r) \geq 0$ so that

$$U^* = \{(x, r) : r \geq 0, \eta_1(r) \leq x \leq \eta_2(r)\}. \quad (6)$$

Furthermore, if A is convex, then U^* is also convex. For $A = A_1$, the functions η_1 and η_2 are given by

$$\eta_1(r) = \begin{cases} 0 & r \leq d \tan(\theta) \\ -d + \cot(\theta)r & r \geq d \tan(\theta) \end{cases} \quad (7)$$

and

$$\eta_2(r) = \begin{cases} 0 & r < r_{\min} \\ \max(0, \eta_1(r), x_c(r)) & r \geq r_{\min} \end{cases} \quad (8a)$$

$$x_c(r) = -d + \sqrt{\left(\frac{r}{c}\right)^{\frac{2}{3}} - r^2}. \quad (8b)$$

The proof of Theorem 1 is given in the Appendix. Note the optimal configuration in Theorem 1 is independent of j . The function $c \mapsto v(U_c)$ is monotonically decreasing on $[0, 2/(3\sqrt{3}d^2)]$. Bisection can be used to find the constant c in Theorem 1 so that $v(U_c) \approx v_{\text{des}}$ [Burden, 1989, Ch. 2.1]. More precisely, bisection will produce a value of c such that $|v(U_c) - v_{\text{des}}| \leq \varepsilon$ in $N = \lceil \log_2(1/(3\sqrt{3}d^2\varepsilon)) \rceil$ volume evaluations. Using (1) and the explicit expressions for η_1 and η_2 given in Theorem 1 to describe (6), we compute the volume of V as

$$v(U) = 2\pi \int_0^\infty (\eta_2(r) - \eta_1(r)) r dr. \quad (9)$$

As a first example, we solve the shape-optimization problem (3) with $A = \mathbb{R}_+^2$, i.e., set $r_{\min} = 0$ and $\theta = \pi/2$ rad in A_1 . In Fig. 2, we plot the shape of U^* for various desired volumes, with all parameters nondimensionalized by d . We see that the cross sections are far from (tapered) rectangles, and they can be physically manufactured by an appropriately designed spool.

IV. COMPARISON OF OPTIMAL GENERAL DESIGN AND OPTIMAL CYLINDRICAL DESIGN

We are interested in quantifying the improvement in field generation with the optimal general electromagnet compared to the optimal cylindrical electromagnet. We can use dimensional analysis to reduce the parameter space for this problem to a minimal set. We are interested in characterizing the relationship between four variables (h {A/m}, j {A/m²}, d {m}, and v {m³}), which are a function of two dimensions (A and m), so we know that the problem can be characterized by $4 - 2 = 2$ dimensionless quantities. Using the Buckingham pi theorem [Hutter 2004], we find that we can express the nondimensional field strength, $hj^{-1}v^{-1/3}$, as a function of the nondimensional distance, $dv^{-1/3}$. We ran a variety of simulations to verify that these

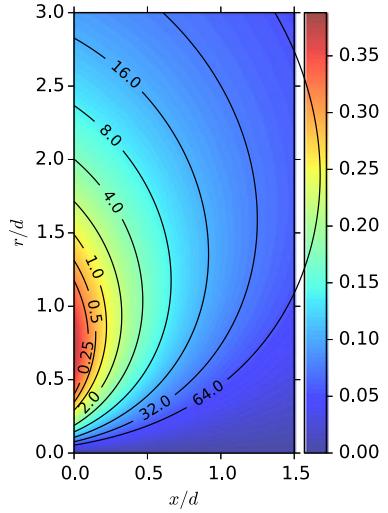


Fig. 2. Illustration of the solution of the shape optimization problem (3), as given in Theorem 1. Here, we take $A = \mathbb{R}_+^2$ (i.e., no r_{\min} or θ constraints). The level sets of $f(x, r) = c$ from (2b) are colored according to the colorbar. The black lines give the solution for various values of the nondimensional volumes, vd^{-3} . As c decreases, the volume enclosed by the corresponding level set increases.

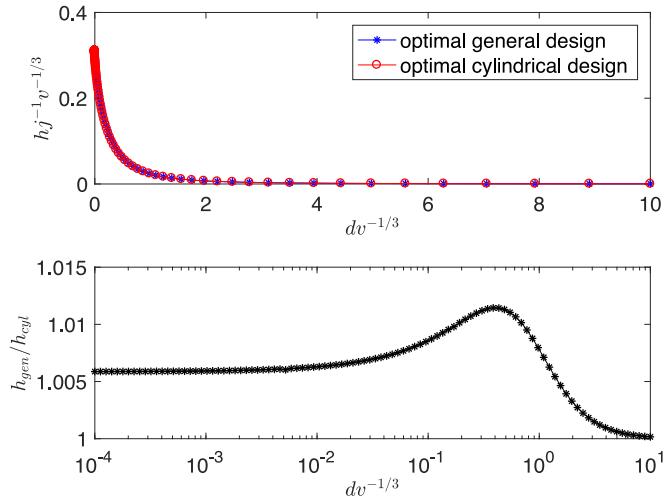


Fig. 3. Numerical experiments showing nondimensional field strength as a function of nondimensional volume for the unconstrained design case (i.e., no constraint on r_{\min} or θ), for both the optimal general design and the optimal cylindrical design. The top plot shows the absolute values of the nondimensional field strengths, and the bottom plot shows the optimal general solution normalized by the optimal cylindrical solution (i.e., the relative improvement in field strength).

two dimensionless terms do, in fact, completely describe the results generally.

Let us begin with the unconstrained case (i.e., $r_{\min} = 0$ and $\theta = \pi/2$ rad). In Fig. 3, we plot the nondimensional field strength as a function of the nondimensional distance for a wide range of values, for both the optimal general design, as well as the optimal cylindrical design. In this nondimensional form, we still observe the $\sim d^{-3}$ decay of field strength that is typical of all magnetic-field sources. Surprisingly, we find that the optimal general design represents only a less than 1.2% improvement over the optimal cylindrical design, over the complete range of parameters.

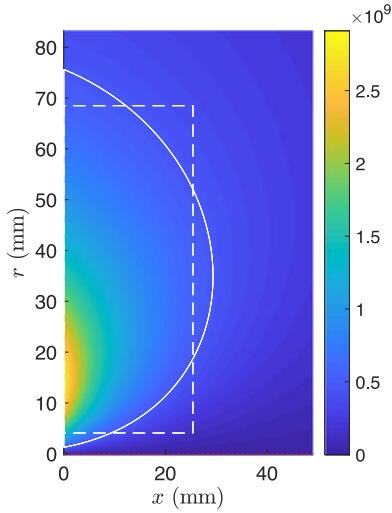


Fig. 4. Comparison of the optimal general (solid white line) and optimal cylindrical (dashed white line) cross sections for the case study with $v = 3.73 \times 10^{-4} \text{ m}^3$, $d = 0.02 \text{ m}$ (i.e., the field is being maximized at the point $(x, r) = (-20, 0) \text{ mm}$), and no additional constraints on r_{\min} or θ .

Let us now consider a real-world case study in which we want to project a field to a distance of $d = 0.02 \text{ m}$. Let us assume an amplifier with a 16 A peak current, and we would like to choose the magnet wire such that the peak current occurs at the peak power output of the power supply. If we assume a 300 W commercial power supply, we would like 16 A to occur at 18.75 V, meaning we would like the resistance of our wire to be 1.17 Ω . Let us assume a 14 AWG copper magnet wire, which has a resistivity of $8.28 \times 10^{-3} \Omega \cdot \text{m}^{-1}$ and a diameter of 1.626 mm (because of the way that wire packs during wrapping, we will assume a square cross section of this dimension, rather than a circular cross section). To achieve our desired resistance, we will need 141 m of wire, for a total volume $v = 3.73 \times 10^{-4} \text{ m}^3$. First, we compute the optimal cylindrical electromagnet to maximize the field at the desired location, using the *fmincon* function in MATLAB. It generates a field at the target location to be $4.717 \times 10^4 \text{ A} \cdot \text{m}^{-1}$, using the maximum current density of $j = 6.05 \times 10^6 \text{ A} \cdot \text{m}^{-2}$. Next, we generate the optimal general electromagnet using the methods of Section III. We compute the maximum field at the target location to be $4.760 \times 10^4 \text{ A} \cdot \text{m}^{-1}$, which represents only a 0.9% increase over the optimal cylindrical electromagnet. To relate these results back to Fig. 3, this optimal general design represents $dv^{-1/3} = 0.278$ and $h_j^{-1} v^{-1/3} = 0.109$. In Fig. 4, we compare the cross sections of the two solutions. It is not surprising that the rectangular cross section of the optimal cylinder appears to be a “best-fit” rectangle to the curved optimal general cross section. From this image alone, it is not obvious that the two cross sections would ultimately result in a magnetic field \mathbf{h} with such negligible differences.

As we consider the addition of constraints, either r_{\min} or θ , it is easy to rationalize that they can only tend to drive the general and cylindrical designs closer together by potentially removing regions that might have been utilized by the general design. As a result, we know that the unconstrained case considered above is the conservative case in which the optimal general design has the best opportunity to outperform the optimal cylindrical design. As a result, we can conclude

that, in all cases, the optimal general design will represent a less than 1.2% improvement over the optimal cylindrical design.

V. CONCLUSION

In this letter, we provided a method to calculate the optimal general geometry of coreless electromagnets, where “optimal” was defined as maximizing the field strength generated at a specific remote point on the axis of the electromagnet for a given volume of conductive material. However, we also found that the optimal general electromagnet negligibly outperforms the optimal (tapered) cylindrical electromagnet for any parameter set, showing a 1.2% improvement at best, and for many parameters substantially less improvement. Because of the added complexity that will come with manufacturing an optimal general electromagnet, designers of magnetic manipulation systems using coreless electromagnets can limit their design space by only considering simple (tapered) cylindrical electromagnets, with confidence that their design will be very close to the optimal general design.

APPENDIX

In this appendix, we provide the proof of Theorem 1. If an optimal solution, U^* , was not a super-level-set of $f(x, r)$, then we could rearrange a small piece of U^* from a position of smaller value of f to a position of larger value of f and increase the value of h , contradicting the optimality.

The maximum of the positive function $f(x, r)$ over \mathbb{R}_+^2 is at $(x, r) = (0, d/\sqrt{2})$ with $f(0, d/\sqrt{2}) = 2/(3\sqrt{3}d^2)$, so this justifies the interval for c .

Since $f(x, r)$ in (2b) is strictly decreasing in x , by rearranging horizontal slices of a set U (with constant r) into intervals as close to the \hat{x} -axis as possible, the value of b increases. If horizontal slices of A are intervals, this implies that each horizontal slice of the optimal shape consists of exactly one interval so that the boundary can then be written as a function, $x(r)$. This justifies the expression for U^* in (6).

The boundary of U^* either coincides with the boundary of A or is a level set of f , satisfying $f(x, r) = c$. Solving for $x = x(r)$ in (2b), we obtain (8b).

Assuming A is convex, U^* is convex since it is the intersection of A and a super-level-set of f , which is a quasi-concave function on \mathbb{R}_+^2 . This can be seen from the fact that $x_c(r)$ in (8b) is a concave function, since it is the composition of $\sqrt{\cdot}$, a concave and increasing function on \mathbb{R}_+ , and $r \mapsto (r/c)^{\frac{2}{3}} - r^2$, a concave function.

Finally, the expressions for η_1 and η_2 for the case $A = A_1$ are obtained by constraining $U \subset A_1$. ■

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